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Annotation

A Note on the Application of Morse Theory to the Study of the Potential Extrema of Body Surface Potential Maps

BY ROBERT GROSSMAN

SUMMARY

The entire body may be approximated with a closed surface having the homotopy type of a sphere. The Morse inequalities then yield certain relationships between the local potential extrema and the saddle points appearing on body surface potential maps (BSPMs). In particular, a list of possible extremal configurations is presented.

Frequently potential extrema appear on isopotential maps of the torso (BSPMs). Pieroni and Narasimhan have studied the movement of these extrema using trajectory maps.[1] Taccardi, Ambroggi and Viganotti[2] have examined their clinical significance, while Rush[3] has attempted to discover their probable source through theoretical considerations. Techniques from Morse theory are applicable to such studies — they simplify the mathematical proofs and lead to new insights.

The part of topology called Morse theory has explored the relationships between the critical points of a function and certain topological invariants that characterize the domain on which it is defined. The inequalities of Morse are a summary of these relationships.[4]

If the entire body is approximated with a closed surface, then it has the homotopy type of a sphere. The Morse inequalities in this case are:

$m_0 \geqslant 1$

$m_1 - m_0 \geqslant -1$

$m_2 - m_1 + m_0 = 2$

where m_0 equals the total number of local potential minimum appearing on the body, m_1 equals the total number of saddle points, and m_2 equals the total number of local maximum. Triples (m_0, m_1, m_2) that satisfy the inequalities are of the following form.

(1, 0, 1)	(2, 1, 1)		(k, k−1, 1)
(1, 1, 2)	(2, 2, 2)	. . .	(k, k, 2) . . .
(1, 2, 3)	(2, 3, 3)		(k, k+1, 3)
.	.		.
.	.		.
.	.		.

This list follows immediately from the inequalities.

Any extremal configuration over the body must satisfy these inequalities, although the converse is not true. That is, some triples (m_0, m_1, m_2) satisfy the inequalities yet could never be produced by actual physiological potential distributions. Note that fixing two of the three numbers m_i determines the third. For example, if one potential maximum and one potential minimum are observed, then precisely one saddle point is possible.

In practice, potentials are usually measured over just the torso and not over the entire body. These missing potentials have not been studied. It is possible that they are either relatively constant or easily interpolated. In any case, the potentials are not measured over a closed surface; rather the potentials for part of a closed surface are measured and the missing potentials are not taken into account. If measurements show that the missing potentials are either constant or accurately interpolable, then the Morse inequalities may be applied to the torso region alone, with the understanding that the critical points may occur on either the measured regions or the interpolated ones. On the other hand, it is possible that the missing potentials contain important information; if this is the case, then these potentials must be measured, and the Morse inequalities must be applied to the entire body.

A more extensive use of Morse theory, when combined with the physiological constraints, would yield further restrictions on the triples (m_0, m_1, m_2) that could actually be realized. Another pertinent application of Morse theory concerns the description of the extrema arising from a finite number of point charges. A paper by Kiang[5] considers such questions. Many other applications are clearly possible.

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Reprint requests to: Robert Grossman, 2903 Huntington Rd., Cleveland, OH 44120.

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